VARIANCE-BASED SENSITIVITY ANALYSIS: NON-PARAMETRIC METHODS FOR WEIGHT OPTIMIZATION IN COMPOSITE INDICATORS

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1. Introduction

Composite indicators are basically computational models used to measure the performance of objects or individuals in complex concepts which are not able to judge based on a single aspect. The role of composite indicators is to provide a proper aggregation that combines the conduct of objects in different dimensions into only one scalar. On the one hand, composite indicators are useful to support decision makers in capturing multidimensional realities and comparing object performance straightforwardly. On the other hand, synthetic indices might provide incorrect benchmarks and misleading policy messages if they are poorly constructed (OECD, 2008; Saisana and Tarantola, 2002).

While the choice of sub-indicators or inputs mainly depends on the definition of the phenomenon, the composite model, including the setting of weights and the aggregation function, is much in the hands of developers. Conceptually, weights refer to the explicit importance of inputs to a composite indicator, and the relative importance (trade-off) between these inputs (OECD, 2008). However, a weight can be directly interpreted as a measure of importance for each input only if several conditions are satisfied: normative weighting, constant variances, and no correlations among variables (Becker *et al.*, 2017). Decancq and Lugo (2013) also pointed out that only under the circumstance of using the weighted arithmetic aggregation and a proper transformation of variables, the ratio of weights becomes equal to the trade-off between input factors. Most composite indicators cannot meet all such requirements, and hence recruiting a measurement of variable importance that is not subject to any model constraints is requisite for weighting.

Given a computational model, there are two main approaches to assess the importance of input variables to the model output: local sensitivity analysis and global sensitivity analysis. This paper focuses on the global approach using variance-based sensitivity measures (Sobol', 1993; Homma and Saltelli, 1996; Saisana *et al.*, 2005). In detail, the article introduces a non-parametric estimation procedure for measuring the importance of input factors, which is developed from the original work of Mara *et al.* (2015) and integrated with the Monte Carlo estimator of Martinez

(2011). The selection of optimal weights is hence carried out by solving a minimization problem in which the weight vector is tuned to achieve the minimum difference between itself and the normalized importance of inputs.

2. Measuring Importance

2.1. Importance Measures for Independent Inputs

Let Y denote a composite indicator obtained from a square integrable function $f(X)$ where the input $X = (X_1, X_2, \dots, X_n)$ is a random vector. Assume that X is defined by a joint probability density function p_X. The variance of the conditional distribution of Y given X_i is denoted by $\mathrm{Var}_{X_{\sim i}}(Y|X_i)$, where the term $X_{\sim i}$ is the vector X without X_i. We can establish a measure of importance for X_i as

$$\mathrm{Var}(Y) - \mathrm{E}\left(\mathrm{Var}_{X_{\sim i}}(Y|X_i)\right) = \mathrm{Var}\left(\mathrm{E}_{X_{\sim i}}(Y|X_i)\right), \qquad (1)$$

which is the expected variance reduction in composite indicator scores if the factor of variation X_i is fixed. According to the ANOVA representation of Sobol' (1993), $f(X)$ can be decomposed into summands of different dimensions:

$$f(X) = f_0 + \sum_{i=1}^{n} f_i(X_i) + \sum_{1 \le i < j \le n} f_{ij}\left(X_i, X_j\right) + \cdots + f_{1\dots n}(X_1, \dots, X_n) \qquad (2)$$

This expression is always existent and unique if the integrals of the summands with respect to any of their own variables are zero (Sobol', 1993). The condition results in all the individual terms in (2) being pairwise orthogonal, implying that all X_i's are mutually independent. The orthogonality leads to the variance decomposition

$$\mathrm{Var}(Y) = \sum_{i=1}^{n} \mathrm{Var}(f_i) + \sum_{1 \le i < j \le n} \mathrm{Var}\left(f_{ij}\right) + \cdots + \mathrm{Var}(f_{1\dots n}). \qquad (3)$$

Sobol' (1993) introduced his measurement of importance, known as Sobol' indices, which is derived from dividing both sides of (3) by $\mathrm{Var}(Y)$ to acquire

$$\sum_{i}^{n} S_i + \sum_{1 \le i < j \le n} S_{ij} + \cdots + S_{12\dots n} = 1, \qquad (4)$$

where

$$S_i = \frac{\mathrm{Var}(f_i)}{\mathrm{Var}(Y)} = \frac{\mathrm{Var}(\mathrm{E}_{X_{\sim i}}(Y|X_i))}{\mathrm{Var}(Y)}, \; S_{ij} = \frac{\mathrm{Var}(f_{ij})}{\mathrm{Var}(Y)} = \frac{\mathrm{Var}(\mathrm{E}_{X_{\sim ij}}(Y|X_i,X_j))}{\mathrm{Var}(Y)} - S_i - S_j, \qquad (1)$$

and so on. The term $X_{\sim ij}$ denotes the vector X without X_i and X_j. S_i is a first-order Sobol' index that captures the main contribution of X_i to the output variance. S_{ij} is a second-order Sobol' index that gauges the contribution caused by the interaction between X_i and X_j, and analogous formulas can be applied to higher-order indices.

Homma and Saltelli (1996) established another measure called a total Sobol' index that captures the total contribution of X_i and all its interactions, defined by

$$ST_i = S_i + \sum_{j \neq i} S_{ij} + \cdots + S_{1 \ldots i \ldots n} = 1 - \frac{\mathrm{Var}(\mathrm{E}_{X_i}(Y|X_{\sim i}))}{\mathrm{Var}(Y)} = \frac{\mathrm{E}(\mathrm{Var}_{X_i}(Y|X_{\sim i}))}{\mathrm{Var}(Y)}. \quad (2)$$

While S_i indicates the expected proportion of variance reduction that would be obtained if X_i was fixed, ST_i indicates the expected proportion of variance that would be left if all inputs were fixed except X_i. Therefore, a large value of either S_i or ST_i implies that X_i is an important contributor and vice versa.

2.2. Importance Measures for Independent Inputs

The application of the Sobol' ANOVA representation to dependent inputs is not prohibited but might lead to incorrect computation and wrong interpretation (Mara and Tarantola, 2012). In Mara *et al.* (2015), the authors proposed a strategy to estimate importance indices that account for the dependency of inputs, using the Rosenblatt (1952) transformation (RT). It transforms $X \sim p_X$ into a random vector $U \sim \mathcal{U}^n(0,1)$ with independent and uniformly distributed entries:

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \\ \vdots \\ X_n \end{bmatrix} \xrightarrow{RT} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_k \\ \vdots \\ U_n \end{bmatrix} = \begin{bmatrix} F_{X_1}(x_1) \\ F_{X_2|X_1}(x_2|x_1) \\ \vdots \\ F_{X_k|X_1,\ldots,X_{k-1}}(x_k|x_1,\ldots,x_{k-1}) \\ \vdots \\ F_{X_n|X_1,\ldots,X_{n-1}}(x_n|x_1,\ldots,x_{n-1}) \end{bmatrix}, \quad (7)$$

where $F_{X_k|X_1,\ldots,X_{k-1}}$ is the cumulative distribution function (CDF) of X_k conditioned by $X_1, \ldots, X_{k-1}$.

Let $U^i = (U_1^i, U_2^i, \ldots, U_n^i)$ be the random vector obtained from RT of X with the order $(X_i, X_{i+1}, \ldots, X_n, X_1, \ldots, X_{i-1})$. Because RT is bijective, there exists an inverse transformation such as $X = RT_i^{-1}(U^i)$ and the aggregation function can be written as $Y = f(RT_i^{-1}(U^i)) = g^i(U^i)$. This establishes a one-to-one mapping

$$(F_{X_i}, F_{X_{i+1}|X_i}, \dots, F_{X_1|X_i,X_{i+1},\dots,X_n}, \dots, F_{X_{i-1}|X_{\sim(i-1)}}) \leftrightarrow (U_1^i, U_2^i, \dots, U_n^i). \tag{8}$$

Since $U_1^i, \dots, U_n^i$ are independent, the variance of Y can be decomposed into the Sobol' indices of U^i instead of X. The first-order index of U_1^i indicates the expected proportion of variance that would be reduced if X_i was fixed and the other factors varied conditionally on X_i. In other words, it quantifies the main contribution of X_i to the output variance, taking into account its dependency with the other inputs. The total index of U_1^i specifies the expected proportion of variance that would remain if all the inputs but X_i were fixed conditionally on X_i, measuring the total dependent contribution of X_i and all its interactions. These two measures are so-called the full Sobol' indices of X_i, denoted by S_i^{full} and ST_i^{full} respectively.

As can be seen from the mapping (8), the first-order index of U_n^i is the main contribution of X_{i-1} that does not account for its dependence on all the other inputs. Therefore, this value is called the independent first-order Sobol' index of X_{i-1}, denoted by S_{i-1}^{ind}. Analogously, the total index of U_n^i is called the independent total Sobol' index of X_{i-1}, denoted by ST_{i-1}^{ind}, that specifies the independent contribution of X_{i-1} and all its interactions. The formulas of full and independent Sobol' indices, and their relationship with the original indices are given as follows:

$$\begin{aligned}
S_i^{full} &= \frac{\mathrm{Var}(\mathrm{E}_{U_{\sim 1}^i}(Y|U_1^i))}{\mathrm{Var}(Y)} = \frac{\mathrm{Var}(\mathrm{E}_{X_{\sim i}}(Y|X_i))}{\mathrm{Var}(Y)} = S_i,\\
ST_i^{full} &= \frac{\mathrm{E}(\mathrm{Var}_{U_1^i}(Y|U_{\sim 1}^i))}{\mathrm{Var}(Y)} = \frac{\mathrm{E}(\mathrm{Var}_{X_i}(Y|(X_{\sim i}|X_i)))}{\mathrm{Var}(Y)},\\
S_i^{ind} &= \frac{\mathrm{Var}(\mathrm{E}_{U_{\sim n}^{i+1}}(Y|U_n^{i+1}))}{\mathrm{Var}(Y)} = \frac{\mathrm{Var}(\mathrm{E}_{X_{\sim i}}(Y|(X_i|X_{\sim i})))}{\mathrm{Var}(Y)},\\
ST_i^{ind} &= \frac{\mathrm{E}(\mathrm{Var}_{U_n^{i+1}}(Y|U_{\sim n}^{i+1}))}{\mathrm{Var}(Y)} = \frac{\mathrm{E}(\mathrm{Var}_{X_i}(Y|X_{\sim i}))}{\mathrm{Var}(Y)} = ST_i.
\end{aligned} \tag{9}$$

In terms of measuring importance, S_i^{ind} points out the expected proportion of variance decline caused by fixing X_i conditionally on all the other inputs while ST_i^{ind} indicates the expected proportion of variance that would remain if all the inputs except X_i were fixed and X_i was set to vary conditionally on them. Overall, a great value of either full or independent Sobol' indices implies that the input factor is important in explaining the variance of composite indicator scores.

3. Estimation Methods and Sampling Strategies

The estimation of full and independent Sobol' indices can be performed using the "pick and freeze" strategy (Saltelli *et al.*, 2008). Only two independent samples of

$U \sim \mathcal{U}^n(0,1)$ with N rows are sufficient to estimate all four importance measures of each input factors. The first step is to generate two random samples $A \sim \mathcal{U}^n(0,1)$ and $B \sim \mathcal{U}^n(0,1)$ with the same size $N \times n$. Then two samples B_1 and B_n are formed by all columns of B except the first (1-st) and the last (n-th) column taken from A respectively. Finally, the indices are calculated using the Martinez (2011) estimator with ρ symbolizing the Pearson correlation coefficients:

$$\begin{aligned} \widehat{S_i^{full}} &= \rho(g^i(A), g^i(B_1)), & \widehat{S_i^{ind}} &= \rho(g^{i+1}(A), g^{i+1}(B_n)), \\ \widehat{ST_i^{full}} &= 1 - \rho(g^i(B), g^i(B_1)), & \widehat{ST_i^{ind}} &= 1 - \rho(g^{i+1}(B), g^{i+1}(B_n)). \end{aligned} \tag{3}$$

Since the composite scores are computed from the samples of U, the inverse Rosenblatt transformation is required to calculate the output $Y = g^i(U^i)$ and the importance indices. If p_X is known, this transformation can be derived from conditional CDFs in X. In practice, p_X is often unidentified and only a representative sample S_X of X is available. The question here is how can we establish a bijective mapping from S_X , which satisfies the property of the inverse Rosenblatt transformation, to provide a sufficiently large number of trials for the Monte Carlo estimation?

A simple solution is to assume a multivariate normal distribution in X then applying Gaussian inverse transform sampling. The distribution parameters Σ and μ can be estimated from S_X, and they in turn are used to construct the conditional inverse CDFs (conditional quantile functions). The second solution for sampling is the Iman-Conover method (Iman and Conover, 1982), which is designed to generate a random sample based on a given correlation structure and known marginal distributions. Because p_X is unknown, the Pearson correlation matrix and the empirical marginals of S_X will be employed instead. The last potential technique is copula sampling based on Sklar's theorem. Having a proper copula model fitted on S_X, one can totally use the inverse copula and empirical marginals to simulate the inverse Rosenblatt transformation.

4. Weight Optimization

With respect to the variable X_i, denote w_i as the weight and I_i as the importance measure using one of the four Sobol' indices. The importance measures for all the variables are normalized by $\widetilde{I_i} = I_i / \sum_{k=1}^{n} I_k$ to make them comparable to the value of weights. Denote a loss function

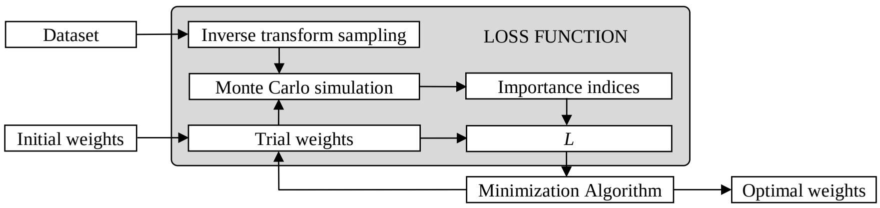
$$L = d^2(w, \tilde{I}) = \sum_{i=1}^{n} \left(w_i - \tilde{I}_i\right)^2, \tag{11}$$

which is the squared Euclidean distance between two vectors $w = (w_1, \dots, w_n)$ and $\tilde{I} = \left(\tilde{I}_1, \dots, \tilde{I}_n\right)$. The optimal set of weights is defined by

$$w^* = \underset{w_1,\dots,w_n}{\operatorname{argmin}} L \quad \text{s.t.} \quad w_i \in (0,1),\ \sum_{i=1}^{n} w_i = 1 \tag{12}$$

At $L_{\min}$, the distance between the two vectors is minimal and hence we attain the set w^* as close as possible to $\tilde{I}$. In case $L_{\min} = 0$ that is equivalent to $w^* \equiv \tilde{I}$, the weights obtained are exactly proportional to the measures of importance. For the general case, L can be always expressed as a function of w and p_X. Thus, we can reach the global minimum if two conditions are satisfied: the joint probability distribution of inputs is given; and the loss function is convex in its domain. Figure 1 gives an illustration of the optimization procedure. At the beginning, a sample of X and an initial set of weights are fed into the loss function to estimate the distance L. The trial weights are then calibrated using a minimization algorithm based on the estimated values of L until the loss function achieves its minimum, which indicates the best course of action.

Figure 1 - *Diagram of the weight optimization procedure.*



5. Empirical Analysis

5.1. Test Case 1: Multivariate Normal Distribution

Considering the composite indicator $Y = w_1X_1 + w_2X_2 + w_3X_3 + w_4X_4$, where $X = (X_1, X_2, X_3, X_4)$ follows a multivariate Gaussian distribution $\mathcal{N}(\mu, \Sigma)$ with the parameters $\mu = (0, 0, 0, 0)$ and

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \sigma_2^2 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \sigma_3^2 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \sigma_4^2 \end{bmatrix} = \begin{bmatrix} 1 & 0.8 & 0.2 & 0.4 \\ 0.8 & 1 & 0.6 & 0.5 \\ 0.2 & 0.6 & 1 & 0.3 \\ 0.4 & 0.5 & 0.3 & 1 \end{bmatrix}. \tag{13}$$

Let ST_i^{full} be the measure of importance. Since the composite model is purely additive, the importance of X_i can be computed as

$$I_i = ST_i^{full} = S_i^{full} = \frac{\mathrm{Var}(\mathrm{E}_{X_{\sim i}}(Y|X_i))}{\mathrm{Var}(Y)} = \frac{\left(w_i + \sum_{j\neq i} w_j \rho_{ji}\right)^2}{\sum_{k=1}^4 w_k^2 + 2\sum_{1\leq p<q\leq 4} w_p w_q \rho_{pq}} \tag{14}$$

which is a single-argument function of w as the correlation coefficients are predefined. Hence, $L = d^2(w, \tilde{I})$ is also a function of w and the optimal weights can be achieved by solving $L = 0$, obtaining $w^* = (0.304, 0.387, 0.143, 0.167)$.

The purpose of this test case is to assess how accurate the weighing procedure could be if only working with the samples of X. Denote $\widehat{w^*}$ as the sample estimate of w^*, the error

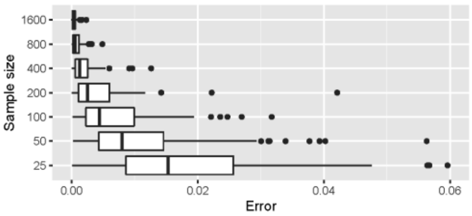
$$E = d^2(w^*, \widehat{w^*}) = \sum_{i=1}^n (w_i^* - \widehat{w_i^*})^2 \tag{15}$$

is a useful gauge to evaluate the similarity between the estimated weights and the true optimal weights. A small E implies that the procedure performs well on the sample, and the expression $\sqrt{E/n}$ measures the average deviation of estimated values from the true ones.

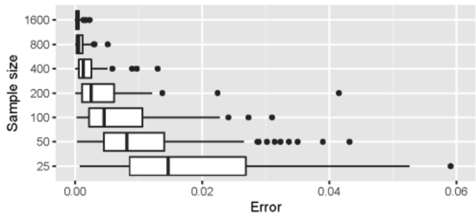
Figures 2a, 2b and 2c describe the boxplots of the error E when applying the procedure with $N = 10^4$ to seven groups of sample sizes, using three sampling methods: Gaussian inverse transform (GIT) sampling, the Iman-Conover method, and Gaussian copula sampling[1]. Each group contains 100 random samples with the same size drawn from $\mathcal{N}(\mu, \Sigma)$. In all three methods, the variation in errors tends to decline as the number of observations in samples increases. At the sample size of 400 onward, we start to acquire sufficiently low and highly stable errors, meaning that the solution derived from samples with more than 400 observations is steady and close to the true optimal weights. Figure 2d illustrates the mean of errors obtained from the three sampling methods. Although there is no clear difference between the techniques across large samples, GIT sampling and the Iman-Conover method seem to outperform Gaussian copula sampling on small samples with less than 100 observations.

[1] The Gaussian copula is chosen among other copulas based on the Akaike information criterion.

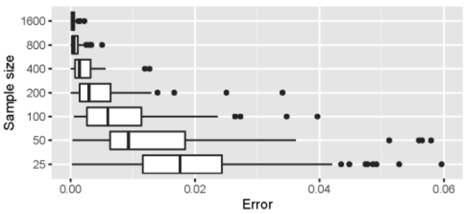
Figure 2 - *Errors by three sampling methods for samples from multivariate normal distribution.*



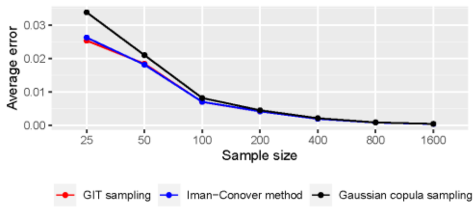
(a) *GIT sampling*



(b) *Iman-Conover method*



(c) *Gaussian copula sampling*



(d) *Average errors*

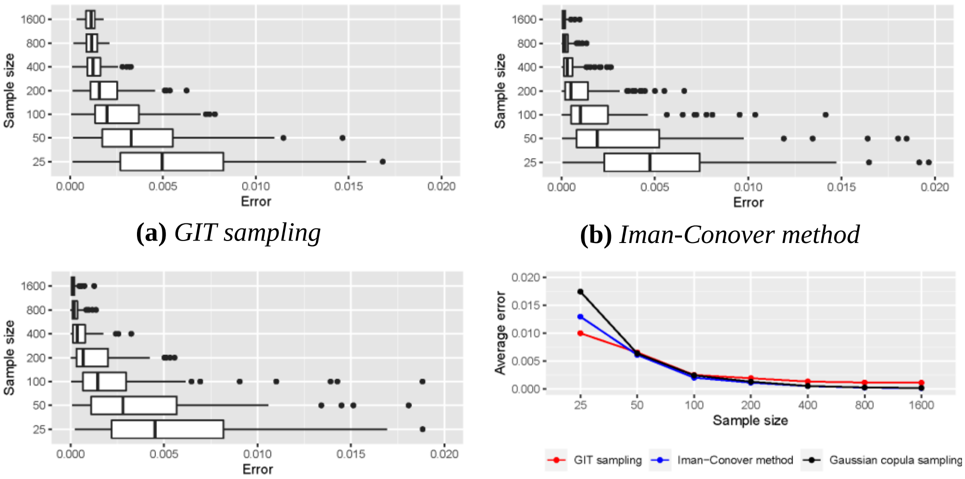
5.2. Test Case 2: Multivariate Mixed Distributions

The second test case uses the same setting as in the first one but performs on multivariate mixed distributions with a more complex model. The composite indicator is defined as $Y = w_1X_1 + w_2X_2 + X_3^{w_3}X_4^{w_4}$, where $X_1, X_2 \sim \mathcal{N}(0,1)$, $X_3 \sim \mathcal{U}(0,1)$, and $X_4 \sim \text{Pois}(4)$. The dependency structure in X is measured using the same correlation matrix as in Equation (13).

In this case, the genuine optimal weights are difficult to calculate directly from distribution parameters because of model complexity and non-normal distributions. An alternative way is employing the inverse Rosenblatt transformation with the true marginal CDFs to produce a huge number of Monte Carlo trials ($N = 10^6$, replicate 1000 times), which in turn is used to estimate an asymptotically true value of w^*. Using this strategy and choosing ST_i^{full} as the measure of importance, the optimal weight is defined as $w^* = (0.243, 0.317, 0.095, 0.345)$.

Figures 3a, 3b and 3c show the variation of error using the optimization procedure with $N = 10^4$ to seven groups of samples. Each group includes 100 equal-sized random samples from the mixed distributions. The accuracy of GIT sampling does not improve after a certain sample size while the other two methods continue lowering the errors toward zero. More evidence of this is shown in Figure 3d, where the average error by GIT sampling seems steady at around 0.001 from the sample size of 400 while the other two methods are constantly improving.

Figure 3 - *Errors by three sampling methods for samples from multivariate mixed distributions.*



(a) *GIT sampling* **(b)** *Iman-Conover method*

(c) *Gaussian copula sampling* **(d)** *Average errors*

5.3. *Practical Case: Human Development Index 2018*

The Human Development Index (HDI) is constructed from three sub-indicators including life expectancy, education, and income, and aggregated by the geometric mean with equal weights. The data for the HDI 2018 used in this section is provided by the UNDP Data Center (https://hdr.undp.org/data-center). Using the Iman-Conover method with $N = 10^4$, the contribution of equal-weighted inputs to the HDI 2018 is given in Table 1. In case ST_i^{full} is selected as the importance index, the total contribution of each input and its interaction, considering its correlation with other inputs, is roughly equal $1/3$. This corresponds to a small loss value, indicating that the original model of the HDI can nearly satisfy the condition of importance weighting based on the full total Sobol' index.

However, if ST_i^{ind} is considered as the importance index, the independent contribution to the output variance is dissimilar between the sub-indicators, leading to a huge loss when comparing the normalized importance indices and the original weights. Since the correlation in the HDI components $\rho = (0.82, 0.84, 0.87)$ is high, one might be interested in a composite indicator that imposes the uncorrelated contribution of each input on the corresponding weight. This indicator can be achieved using the optimization procedure based on the independent total Sobol' index. The optimal weights from the Iman-Conover method with $N = 10^4$ are $\widehat{w^*} = (0.584, 0.177, 0.239)$ for life expectancy, education, and income.

Table 1 - *Importance measures for the HDI components in 2018 using equal weights*.

	Life expectancy	Education	Income	L
Normalized $\widehat{ST}_i^{full}$	0.312	0.344	0.344	0.0007
Normalized $\widehat{ST}_i^{ind}$	0.150	0.487	0.363	0.0580

Figure 4 - *Greatest shifts in the HDI ranking using the optimized weights based on ST_i^{ind}.*

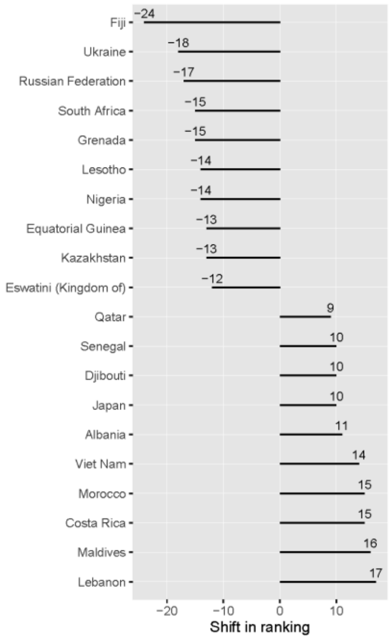


Table 2 - *Ten countries with the highest and lowest HDI rankings using the original weights and the optimized weights.*

Rank	Equal weights	Optimized weights
1	Norway	Hong Kong (+5)
2	Switzerland	Switzerland (+0)
3	Ireland	Norway (−2)
4	Germany	Singapore (+8)
5	Iceland	Australia (+2)
6	Hong Kong	Iceland (−1)
7	Australia	Ireland (−4)
8	Sweden	Sweden (+0)
9	Netherlands	Netherlands (+0)
10	Denmark	Japan (+10)
180	Eritrea	Guinea-Bissau (−2)
181	Mozambique	Burkina Faso (+2)
182	Sierra Leone	Mozambique (−1)
183	Burkina Faso	Mali (+2)
184	Burundi	Burundi (+0)
185	Mali	South Sudan (+1)
186	South Sudan	Niger (+3)
187	Chad	Sierra Leone (−5)
188	CAR	Chad (−1)
189	Niger	CAR (−1)

Note: The numbers in parentheses denote the place changes from the original ranking.

Figure 4 describes the most increases and declines in the HDI ranking when applying the optimized weights compared to the original weights. The countries that have the highest promotion in ranking are Lebanon and Maldives while Fiji, Ukraine, and Russia occur the most ranking reductions. Table 2 compares the proportion of ten countries with the highest and lowest rankings on the original HDI table with the same proportion derived from the new index. In the upper part, the positions of Switzerland, Sweden, and the Netherlands remain unchanged. Hong Kong jumps from sixth place to first place while Singapore and Japan make a significant leap to present in the top ten countries. In the lower part, despite several slight disturbances in positions, the bottom ten countries are quite similar between the two ranking tables.

6. Conclusion

This paper introduces a new weighting method for composite indicators based on a measure of importance and Monte Carlo simulations. The full and independent Sobol' indices (Mara *et al.*, 2015) and the Martinez estimator (Martinez, 2011) are two key factors used to establish a complete procedure for optimizing weights given a sample of inputs and a predefined aggregation model. The procedure allows developers to obtain a solution in which the magnitude of weights coincides with the dependent or independent contribution of inputs to the variance of composite scores. The method can be widely applied to all composite models since it works with any single-valued function regardless of complexity.

During the optimization procedure, sampling strategies play a vital role in the precision of estimation results. Three sampling techniques were tested on different data structures and model configurations. Gaussian inverse transform sampling is the simplest approach, but it is only suitable for data from the multivariate normal distribution. The Iman-Conover technique and copula sampling show greater effectiveness as they can handle samples from mixed distributions and produce near-maximum accuracy with a sufficiently large sample size. In the case of small sample sizes, checking for outliers before sampling is required because they might distort the simulation and result in inaccurate estimates of importance indices.

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SUMMARY

This paper presents an optimization procedure that helps composite indicator developers achieve the most plausible choice of weights without being restricted as the complexity of synthetic models escalates. Given a predefined aggregation function, variance-based sensitivity analysis and Monte Carlo simulations are employed to establish non-parametric methods for measuring the importance of each input to the output uncertainty. Utilizing the computational power of these methods, the weights are calibrated by an optimization procedure to attain the best fit with the estimated measures of importance. The procedure has been tested in two artificially created examples and in one practical case of well-being measurement to confirm its accuracy and efficiency in building composite indicators.

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