WELL-BEING: A BETA-DISTRIBUTION-BASED APPROACH

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Abstract. In recent years, there has been growing attention to evaluating well-being at local level. Along this line, since 2013, the Italian Institute of Statistics (ISTAT) annually releases a dashboard of indicators to measure the so-called Equitable and Sustainable Well-Being (BES) for Italy, its Macro-area (NUTS-1) and regions (NUTS-2). More recently, the ISTAT provides BES indicators at local level (NUTS-3), related the 107 Italian provinces and metropolitan cities.
The aim of the paper is to provide a more in-depth analysis of territorial inequalities and divergences across the Italian provinces. Specifically, the paper represents the first attempt to synthetize the main domain of BES (economic, social and environmental among others) through the parameters underlying the Beta distribution of the multidimensional well-being. The parameters - mode and concentration- associated with the Beta distribution and used as a proxy of territorial disparities identify a high degree of heterogeneity not only between the Northern and Southern Italian provinces, but also among adjacent provinces.

1. Introduction

In recent years, there has been considerable interest among researchers and policymakers in assessing well-being. Traditionally, per capita Gross Domestic Product (GDP) has been used as an indicator of societal well-being, but it has limitations as it mainly measures the economic aspect of a country and does not fully capture overall welfare. Initiatives like the European Commission's "Going beyond GDP" and influential reports (Kolm, 1977; Atkinson *et al*., 1982; Stiglitz, 2009) have highlighted that income alone cannot adequately represent the complexity of well-being.

Research has shown that well-being includes more than just material wealth, encompassing subjective elements such as perceptions of living standards (Ivaldi *et al.*, 2016; Bleys *et al*., 2012; Noll, 2002; Sen, 1980).

As a result, it is widely recognized that measuring well-being requires consideration of both monetary and non-monetary factors.

For instance, the OECD suggests that well-being assessments should cover aspects like employment, housing, health, work-life balance, education, social connections, civic engagement, governance, environment, personal security, and subjective well-being.

Well-being is essentially a “complex system” composed of numerous components (Greco *et al.*, 2019). In the literature, two approaches address the multidimensionality of well-being: the composite index approach and the dashboard of indicators approach (see Hoffmann *et al*., 2008).

While, a dashboard provides a detailed array of single indicators across various dimensions of well-being, a composite index consolidates information from several dimensions into a single value.

The benefits of composite indices are clear as they provide a unidimensional measurement of well-being (Mazziotta and Pareto, 2016). True to its nature, a composite index is usually built to ‘tell a story’. It is, thus, ideally suited to identify and bring attention to a possibly latent phenomenon (Kuc-Czarnecka *et al*., 2020).

In this context, following Polinesi *et al.* (2024) we consider the multidimensional well-being as a Beta-distributed random variable within the interval (0,1), characterized by unknown parameters α and β.

It is well known that concave beta distributions, with shape parameters greater than 1, can be parametrized in terms of mode and concentration. We use this parametrization to compute a non-compensatory composite indicator (see Mazziotta and Pareto, 2018 for details).

The composite indicator is calculated for each province and for the years 2019 and 2022, capturing changes in multidimensional well-being over time, particularly before and after the onset of the Covid-19 pandemic.

The aim is to deepen understanding of the spatial distribution of the multidimensional well-being index at the local level in Italy, with a focus on Italian provinces. This insight can assist policymakers in directing resources to the most disadvantaged areas.

Moreover, we introduce a new measure, the Bivariate Beta Distribution Impact (BBDI for short) measure. This metric evaluates the contribution of individual regions (or provinces) to overall well-being, offering a new perspective on local disparities.

These two levels of analysis reveal that the traditional Italian North vs. South divide is clear.

The proposed approach lies within the framework of composite indicator representing a novelty in the literature, since to the best of our knowledge, this is the first time that multidimensional well-being is evaluated by means of a family of probability function.

Indeed, few papers on applying beta distribution focus exclusively on poverty and inequality measures (see for example Chotikapanich *et al*., 2012; Anderson *et al*., 2014; De Nicolò *et al.*, 2024).

The remainder of the paper is organized as follows. Section 2 introduces data and methodology used to construct the composite index of well-being for Italian provinces. It also discusses the measure to evaluate the regional contribution on Italian well-being. Section 3 illustrates the empirical results. Finally, Section 4 draws some conclusions.

Table 1 – *List of the selected elementary indicators and their corresponding pillar*.

Pillar	**Indicator**	**Unit**
Economic well-being	Employment rate (20-64 years old)	Percentage
	Youth employment rate (15-29 years old)	Percentage
	Average annual per capita income (pensions)	Euro
	Mobility of graduates (25-39 years old)	For 1,000 inhab.
	Non-partecipation rate	Percentage
Social well-being	Life expectancy at birth	Years
	Children who benefted of early childhood services	Percentage
	Participation in the school system of children	Percentage
	People with at least upper secondary education	Percentage
	People having completed tertiary education	Percentage
	Transition to university	Cohort specific rate
	Participation in long-life learning	Percentage
	Non-profit organizations	For 10,000 inhab.
	Women’s political representation in municipalities	Percentage
	Young people’s political representation	Percentage
	Municipalities: collection capacity	Percentage
	Hospital beds in high-care wards	For 10,000 inhab.
	Employees in cultural enterprises	Percentage
	Public transport network	Seat-*km* per capita
	Hospital beds	For 10,000 inhabitants
	Age-standardized avoidable mortality rate	For 10,000 inhab.
	Age-standardised cancer mortality rate	For 10,000 inhab.
	Age-standardised mortality rate for dementia	For 10,000 inhab.
	NEET	Percentage
	Inadequate numerical competence	Percentage
	Inadequate literacy skills competence	Percentage
Environmental well-being	Availability of urban greenery	m^2 per capita
	Municipal waste separately collected	Percentage
	Electricity from renewable sources	Percentage
	Landfll of urban waste	Percentage
	Consecutive dry days	Days
	Municipal waste produced	*Kg* per capita
	Density and importance of musems’ heritage	Per 100 km^2
	Agritourism businesses	Per 100 km^2
	Density of historic greenery	Per 100 m^2

2. Data and methods

In 2022, as part of the "BES at local level" project, ISTAT published a set of 70 elementary indicators across 11 domains to describe the well-being of the 107 Italian provinces.

For our study, we use available data from the years 2019 and 2022 to monitor the well-being of these territories and belonging region over time. Specifically, the analysis involves 40 elementary indicators described in Table 1.

Let us go into the details of our framework. We consider a complex phenomenon and a cross-sectional dataset of variables (or indicators), $X_1, X_2, \ldots, X_k$, observable over a population of n units. We denote the values of the variables observed across the n units as x^i_j, where i = 1, 2, …, n and j = 1, 2, ..., k.

We model the multidimensional well-being as a Beta distributed random variable with support in the interval (0,1). To this end, we normalize the variables x^i_j according to the min-max approach.

Specifically, these scaled variables are defined as:

$$z_j^i = \frac{x_j^i - \min\limits_i\left(x_j^i\right)}{\underset{i}{\mathrm{m}}\left(x_j^i\right) - \min\limits_i\left(x_j^i\right)}, i = 1,2,\ldots,n,\ j = 1,2,\ldots,k. \quad (1)$$

If the j-th indicator has negative polarity, the complement of z_j^i with respect to 1 is computed[1]. This normalization procedure ensures that all indicators are positively correlated with the phenomenon we aim to measure.

Let z_j^i the realizations of $Z \sim Beta(\alpha, \beta)$, where $Beta(\alpha, \beta)$ denotes the Beta distribution of unknown parameters α, β > 0. The probability density function of Z is:

$$f(z, \alpha, \beta) = \frac{z^{\alpha-1}(1-z)^{\beta-1}}{B(\alpha,\beta)},\ 0 < z < 1, \quad (2)$$

where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha,\beta)}$ and Γ is the Gamma function.

The shape of the Beta distribution varies with parameters α and β. For example, when both parameters are greater than 1, the distribution can be reparametrized in terms of the mode ω (i.e., the most likely value of the distribution) and concentration κ (i.e., absence of variability). Formally:

[1] The polarity represents the sign of the relation between the indicator and the phenomenon to be measured. We have a positive polarity if the individual indicator represents a dimension considered positive, that is, increasing variations of the indicator correspond to positive variations of the phenomenon. Similarly, we have a negative polarity if it represents a dimension considered negative.

$$0 < \omega = \frac{\alpha-1}{\alpha+\beta-2} < 1 \tag{3}$$

$$2 < k = \alpha + \beta < +\infty \tag{4}$$

This parameterization provides a more intuitive understanding compared to the original parameters[2].

The parameters of the well-being distribution are estimated at unit level, by applying the maximum likelihood approach to the scaled variables associated z_j^i, and the parameters of well-being α and β are then obtained by computing the sample mean of all estimates at unit levels.

Consequently, we have:

$$\alpha = \frac{1}{n}\sum_{i=1}^{n} \alpha_i, \tag{5}$$

$$\beta = \frac{1}{n}\sum_{i=1}^{n} \beta_i. \tag{6}$$

Thus, the unit-level parameters are weighted averages of their population-level counterparts. Notably, values of α and β estimated in Eqs. (5) and (6) closely align with those obtained through the maximum likelihood approach at the overall level, ensuring the robustness of the results[3].

In the following analysis, we examine variations in both mode and concentration by individually excluding each unit to assess its impact on well-being across Italy.
Specifically, we first estimate the parameters α and β of the Beta distribution associated with the overall population made by all units, then we estimate the parameters α_i^* and β_i^* of the Beta distribution associated with the population obtained eliminating the i-th unit from the overall population.
Therefore, the Bidimensional Beta Distribution Impact measure (BBDI measure for short) for each unit i can be defined as the two-dimensional vector containing the variation associated with mode and concentration, as follows:

$$\left[\frac{\omega-\omega_i^*}{\omega_i^*}; \frac{k-k_i^*}{k_i^*}\right], \quad i=1,2,\ldots,n, \tag{7}$$

where ω_i^* and k_i^* are computed using Eqs. (5), (6) with α_i^* and β_i^*.

[2] We refer to Nadarajah and Kotz (2007) for a review of the properties and the variations of Beta distributions as well as their relationship to other distributions.
[3] The difference between the parameters obtained through sample mean and maximum likelihood estimation is 0.06 for the value of α (1.75 vs 1.81) and 0.02 for the value of β (1.50 vs 1.52).

Then, the non-compensatory composite index of well-being is computed for the units over the study period. Formally, the composite index for the i-th unit is defined as:

$$I_i = \omega_i - \frac{1}{k_i}, \quad \text{i=,2,...,n,} \tag{8}$$

where $\frac{1}{k_i}$ represents the penalty due to the dispersion level, i.e. the extent to which the values of the elementary indicators deviate from the mode. Notably, as $k_i \to \infty$ (indicating maximum concentration) the value of the index depends solely on ω_i.

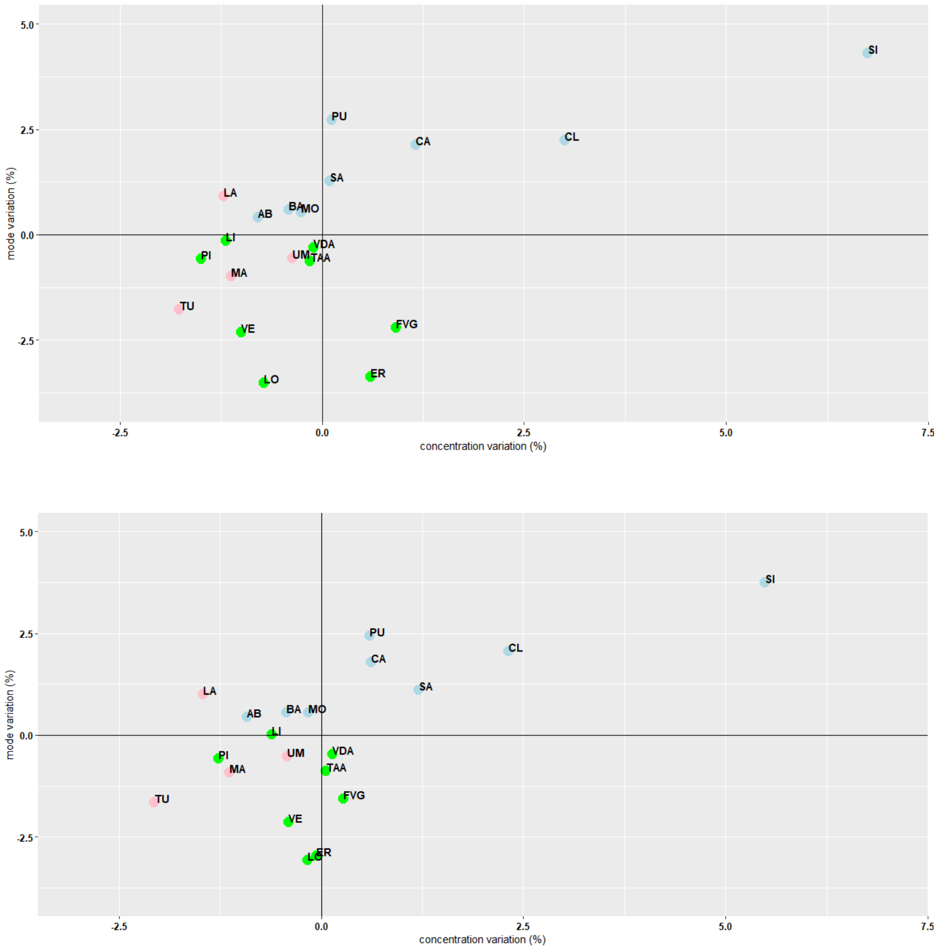
The quantity in (5) ranges between -1/2 and 1. It assigns -1/2 to the unit i associated with minimum concentration, k_i = 2, and mode equal 0 (i.e., the worst case) and 1 when the concentration is maximum, $k_i \to \infty$, and mode equal 1 (i.e., the best case).

Therefore, Eq. (8) as outlined in Mazziotta and Pareto (2016) decomposes the score of each i-th unit in two parts: mode level (ω_i) and penalty $\left(\frac{1}{k_i}\right)$. The penalty is a function of the indicators' dispersion in relation to the mode value and it is used to penalize the units. The aim is to reward units that, while having the same mode, exhibit greater balance among the indicator values.

3. Results

In this section, we present the results for changes in mode and concentration, as well as the well-being composite index defined in Eq. (8), separately for the two years considered. First, following Eq. (7), we compute the variation in mode and concentration at the regional level (Figure 1) and the variation in mode at the provincial level (Figure 2). Finally, we illustrate the spatial distribution of the well-being index across provinces (Figure 3).

Figure 1 – *Mode and concentration variation by eliminating each region at a time for the years 2019 (top) and 2022 (bottom). Colours indicate Italian macro-areas: South (light-blue), pink (Centre) and green (North).*



Note from Figure 1 that the Southern provinces - Puglia, Sardinia, Campania, Calabria and Sicily - if removed from the analysis of multidimensional well-being, would lead to a concentration of Italian multidimensional well-being at higher levels as they are associated with positive variation of mode and concentration (first quadrant). Conversely, Northern provinces contribute positively to Italian well-being (fourth quadrant). For simplicity, relatively to provinces we show only the mode variation (Figure 2).

Figure 2 – *Spatial distribution of mode variation for the years 2019 (left) and 2022 (right). Green (red) colour indicate positive (negative) impact on Italian well-being.*

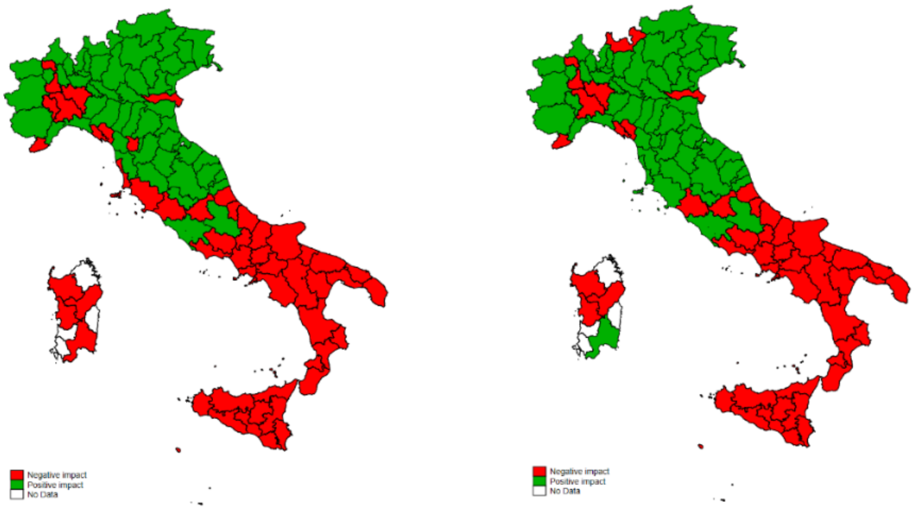


Figure 2 highlights that, from 2019 to 2022, the number of provinces experiencing a positive variation in mode increases. However, a clear divide between the North and South persists, with some exceptions observed in Lombardy.

Figure 3 – *Spatial distribution of well-being for the years 2019 (left) and 2022 (right). Darker colors indicate higher level of well-being.*

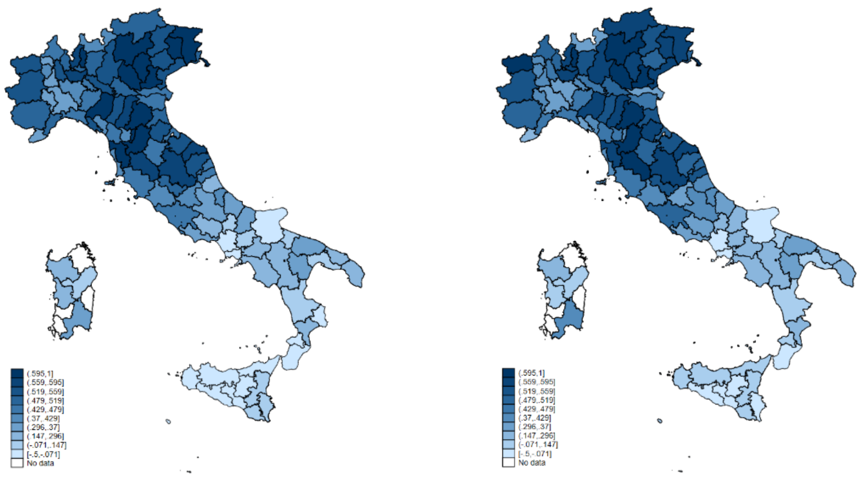


Figure 3 illustrates a geographical representation of how the multidimensional well-being index is distributed across the Italian provinces over the years 2019 and 2022. Its values are plotted following a colour scale with darker colours representing higher levels of well-being. A distinct divide between northern and southern provinces is evident, with a few notable exceptions.

Overall, the well-being index shows a slight increase in 2022 compared to 2019, potentially reflecting the impact of social protection policies aimed at supporting the most vulnerable populations during the pandemic (Polinesi *et al.*, 2023).

Table 2 – *Ranking of Italian provinces based on the value of the composite index for the years 2019 and 2022 with brackets indicating the change in position over time.*

Worst ten		Top ten	
2019	2022	2019	2022
Crotone	Crotone	Trieste	Trieste
Enna	Enna	Firenze	Firenze
Agrigento	Sud Sarde (92)	Prato (54)	Trento
Napoli	Agrigento	Bologna	Lecco
Caltanissetta	Foggia (96)	Pordenone (14)	Padova
Trapani (94)	Napoli	Lecco	Bologna
Vibo Valentia (89)	Caltanissetta	Verona	Verona
Messina (93)	Siracusa (95)	Parma (15)	Aosta (24)
Palermo (97)	Reggio Calabria	Trento	Siena (14)
Reggio Calabria	Caserta (97)	Padova	Udine (12)

Table 2 presents the rankings of the ten best and worst provinces based on their well-being levels, highlighting a clear divide between the North and South. In the lower part of the rankings, despite minor changes in positions, the bottom ten provinces remain largely consistent across the two years considered.

Conducting a territorial analysis of well-being is crucial for identifying the geographical areas most in need and for better directing available economic resources.

4. Conclusions

This paper contributes to the analysis of well-being across Italian provinces and regions by modeling multidimensional well-being as a Beta-distributed random variable. It develops a multidimensional index to track well-being over time, focusing on two key measures: mode and concentration.

The findings indicate that Southern provinces experience greater losses in well-being, as reflected by lower composite index values, and that Southern regions have a negative impact on overall Italian well-being.

While the aggregate index reveals no significant differences in well-being levels before and after the pandemic, further analysis of individual pillars or specific indicators within these pillars could yield different insights. Future research could explore these dimensions in greater detail.

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