LONGEVITY-RISK-ADJUSTED GLOBAL AGE AS A MEASURE OF WELL-BEING

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1. Introduction

Survival analysis refers to a set of statistical techniques used to estimate the probability of the occurrence of a given event as a function of time (Collet, 2003). This collection of statistical procedures is also known as time-to-event analysis. It focuses on the study of the time elapsed from a specific moment up to the point when a specific event (death) will occur.
Models underlying survival analysis are usually expressed in terms of age-dependent mortality rate estimates.

Two types of approach emerge from the literature: a parametric approach that specifies a mathematical form for mortality rates (hazard rates) as a function of chronological age, and a non-parametric approach that sets the mortality rate of a given age as a weighted average of the 'raw' mortality rates.

According to the parametric approach, the seminal work of Gompertz (1825) implies that the total death rate is composed of an age-dependent term which increases exponentially with chronological age.

A first and non-exhaustive revised version of Makeham (1867) includes a non-age-dependent mortality rate in the Gompertz mortality law. In fact, the Gompertz–Makeham (GM) model has been expanded several times to allow for a specific shape of mortality. For example, Heligman and Pollard (1980) model mortality rate as an eight-parameter function of age, specifying the model in terms of mortality odds. Kostaki (1992) extends the Heligman-Pollard model to eliminate some biases in its estimation. Furthermore, Dellaportas *et al.* (2001) estimate the Heligman–Pollard model within a Bayesian framework.

Tabeau *et al.* (2002) review parametric functions used to model mortality and discuss the applicability of different parametric models to estimate the age and effects of mortality.

Recently, Milevsky (2020) developed a computational framework for inverting GM mortality hazard rates to present a new definition of age: the longevity-risk-adjusted global (L-RaG) age. The computation of longevity-risk-adjusted global age involves multifaceted aspects of aging by allowing the estimation of the number of years a person seems to be (biological age) from his/her chronological age. Indeed,

the computational process of L-Rag age begins by collecting data on mortality rates as a function of chronological age, as well as any other characteristics or elements associated with mortality.

On the other hand, non-parametric models used to estimate the survival function from lifetime data do not assume a theoretical distribution for *F*. This approach is inherent to studies of clinical trials where the following estimators assume particular importance: Kaplan-Maier estimator (Kaplan *et al.*, 1958), Nelson–Aalen estimator (Nelson, 1969), and longrank test (Peto and Peto, 1972).

We contribute to the existing literature by applying the above-mentioned work of Milevsky (2020) to obtain L-RaG age estimates among Italian regions for two different years (2011 and 2018), while proving that this quantitative indicator of biological age is related to a sentiment indicator of the perceived age (healthy life expectancy and one's life satisfaction) provided by the ISTAT database within the BES project[1]. The ISTAT indicators are obtained via surveys with the aim of detecting living conditions in Italian regions and reflecting human feeling. An interesting point is that an objective indicator (i.e., L-RaG age) can mimic a sentiment indicator, thus implying that human feeling relies on different lifestyle conditions. Analysis at regional level allows us to investigate the impact of the well-known difference between Northern and Southern Italy considering a biological indicator on different age classes.

In the following sections, we describe the GM mortality law and L-RaG age indicator, and we conclude with some results.

2. The GM mortality law

The GM mortality law is the linear relationship between the natural mortality rate and chronological age *x*, i.e., the number of years a person has been alive. Indeed, every adult life in the region *i*, for $\mathrm{i} = 1, \ldots, \mathrm{N}$ can be expressed by the following parametrization:

$$\mu_x[i] - \lambda[i] = \begin{cases} h[i]e^{g[i]x} & x < x^* \\ \lambda^* & x \geq x^* \end{cases}, \tag{1}$$

where $\mu_x[i] = 1/(1 - q_x[i])$ is the total hazard rate (THR) and $q_x[i]$ obtained from ISTAT mortality tables is the one-year decrement rate.

The region-specific parameters $\lambda[i],\ h[i], g[i]$ represent the accidental death rate, a.k.a. the Makeham constant, the initial natural mortality rate (INMR), and the

[1] Data collected from a sample of respondents that took a survey.

mortality growth rate (MGR), respectively. As well, x^*, i.e., the critical chronological age at which the Gompertzian regime ends, and $\lambda^* > \lambda[i]$, i.e., the plateau mortality rate faced when chronological ages are equal to x^*, are global parameters estimated at the global (Italian) level.

Parameters λ^* and x^* are also known as species-specific accidental mortality rate the species-specific lifespan (Richards, 2020).

Rearranging Eq. (1), the model can be expressed as:

$$ln(\mu_x[i] - \lambda[i]) = ln(h[i]) + g[i]x, \quad x < x^* \tag{2}$$

which is the standard linear representation of total hazard minus accidental death rates for all ages within the GM regime $[x, x^*]$.

The relationship can be explicitly written as:

$$\overbrace{ln\left(ln\left(\frac{1}{1-q_x[i]}\right) - \lambda[i]\right)}^{z} = \overbrace{ln(h[i]) + ln\left[(e^{g[i]x} - 1)/g[i]\right]}^{K_0} + \overbrace{g[i]}^{K_1} x \tag{3}$$

to estimate the region-specific GM parameters $\lambda[i], h[i], g[i]$, where the new constants K_0 and K_1 are defined for convenience, see Milevsky (2020). Global parameters x^* and λ^* are estimated via regression through the equation:

$$ln(h[i]) = L + (-x^*)g[i] + \varepsilon_j, \tag{4}$$

where $L = ln\lambda^*$.

The GM mortality law implies that (log) mortality rates increase linearly and then converge to a constant mortality plateau, which is also known as the compensation law. This leads to a linear negative relationship between the initial natural mortality (intercept) $ln(h[i])$ and MGR (slope) $g[i]$ as shown in Eq. (4).

The next Section focuses on the computation of the L-RaG age starting from the GM equation, using local and global mortality rates as input.

2.1 From GM mortality law to L-RaG age

According to the work of Milevsky (2020), *ξ(x,i)* denote the longevity-risk-adjusted global age for someone at the chronological age of *x* in region *i*, which may or may not correspond to his/her chronological age (x).

The L-RaG refers to the concept of biological age, i.e., the age (number of years) a person seems to be, whose computation involves mapping from mortality rates to a specific age by inverting the GM mortality law.

For this reason, the L-RaG age is forced to satisfy the compensation law described in Eq. (1), but considering the Italian average version of the GM parameters. Formally, we have:

$$ln[\Lambda_\xi[i] - \Lambda] = ln[H] + G\xi(x,i), \tag{5}$$

where $\Lambda_\xi[i]$ is the longevity-risk-adjusted global hazard rate and $\Lambda \geq 0, H > 0, G \geq 0$ represent the mean in Italy of $\lambda[i], h[i], g[i]$, respectively.

The longevity-risk-adjusted global hazard rate is set equal to the THR:

$$\Lambda_\xi[i] = \mu_x[i]. \tag{6}$$

Inverting the GM equation and solving by $\xi(x,i)$, the L-RaG age can be determined:

$$\xi(x,i) = \frac{ln[\lambda[i] - \Lambda + h[i]e^{g[i]x}] - ln[H]}{G}. \tag{7}$$

The validity of $\xi(x,i)$ is tied to $\lambda_i - \Lambda + h_i e^{g[i]x} > 0$. This fact is generally guaranteed because the GM model is applied to adult ages (35 and older).

Eq. (7) is used to estimate the L-RaG age in the Italian region. The data and results are discussed in Section 3.

3. Data and results

We consider data from ISTAT mortality tables for Italian regions considering for two years (2011 and 2018) males and females separately.

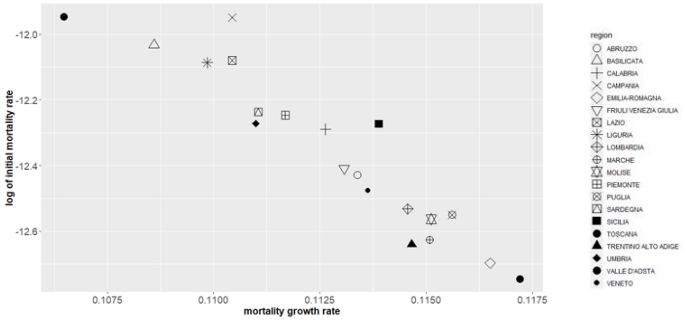
First of all, we extract the one-year decrement rate $q_x[i]$, then we estimate all the GM parameters as described in Section 2.

The compensation law is assured for all subgroups and years analyzed for values of x^* and λ^* equal to 95 and 0.06. The negative relationship between mortality growth rate and (log) initial mortality rate is shown in Figure 1.

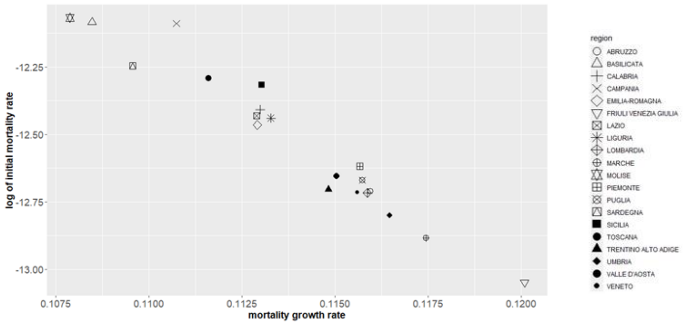
Figure 2 shows chronological ages versus the gap (difference) between chronological age and L-RaG age for females (a) and males (b) in each region considered.

Figura 1 – *Compensation law: females 2011-2018 (a, b) and males 2011-2018 (c, d).*

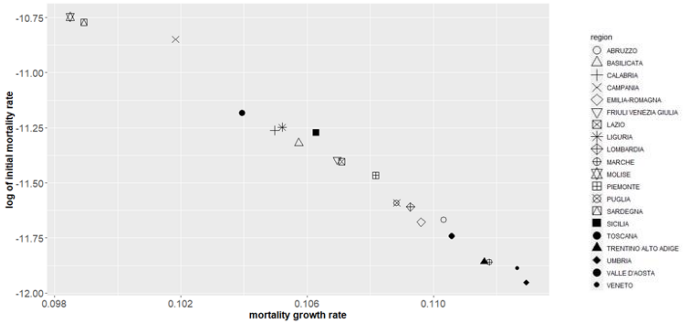
(a)



(b)



(c)



(d)

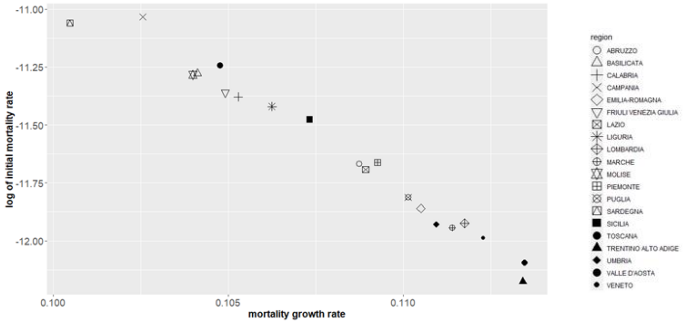
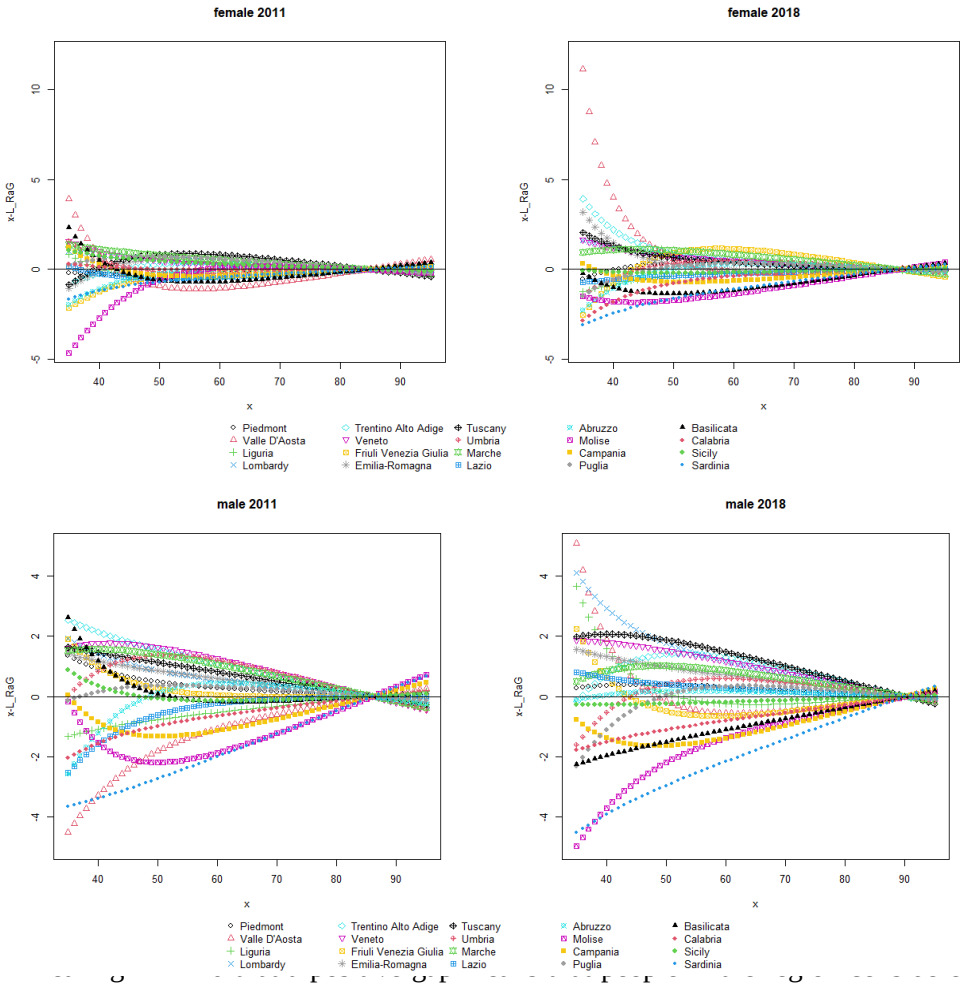
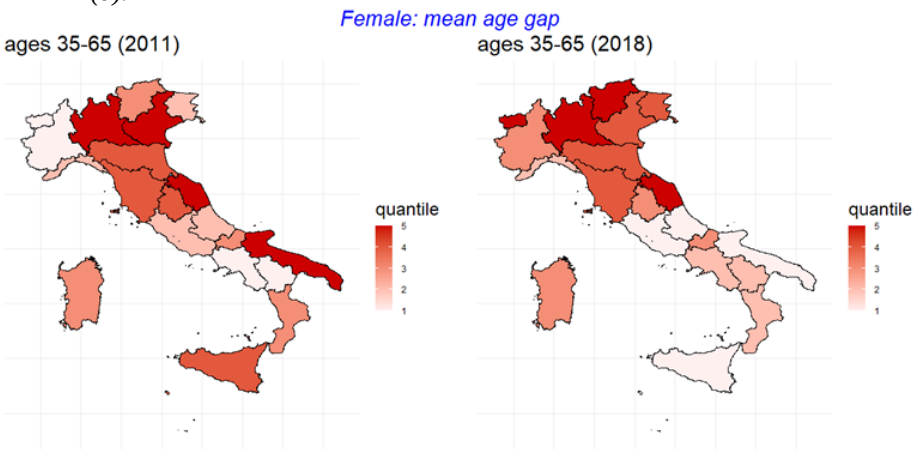


Figure 2 – *Relationship between chronological age (x) and the mean age gap (x-L-RaG) at regional level.*

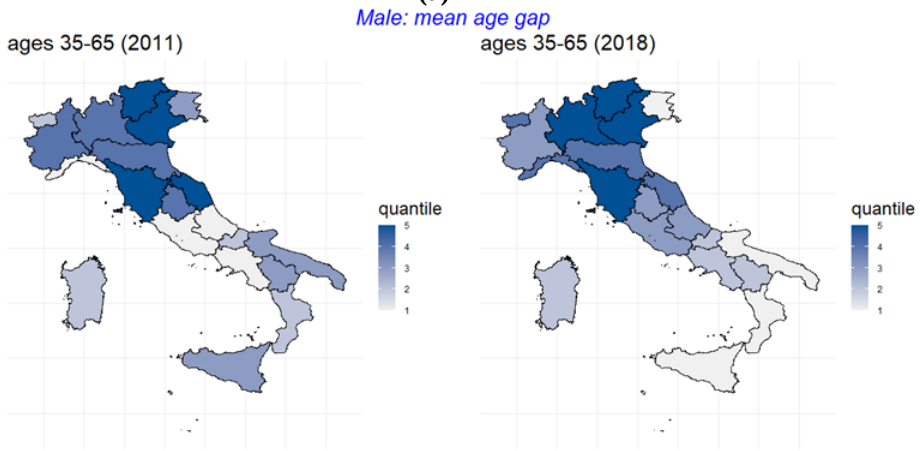


seem younger than their effective age, Figure 3 focuses on the mean age gap at the regional level, divided into quantiles relatived to the adult category (ages 35-65).

Figure 3 – *Mean age gap at regional level divided by quantiles for females (a) and males (b).*



(a)



(b)

Figure 3 shows how the situation worsened on average from 2011 to 2018 for adults (females and males) in the Southern regions (quantiles are lower) marking a distinction between Southern and Northern Italy in terms of the difference between chronological age and L-RaG age.

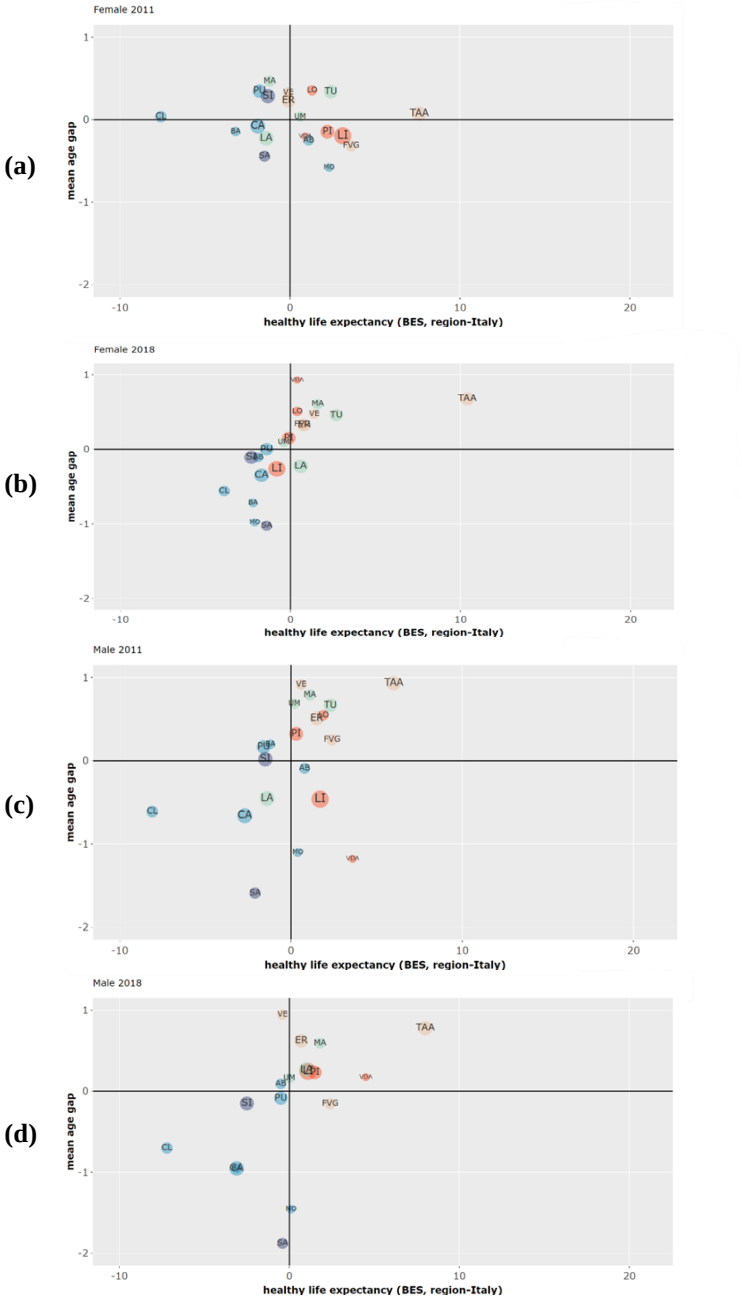
To investigate the age gap in terms of sentiment indicators, we relate the mean age gap (*x*-L-RaG) to healthy life expectancy and personal life satisfaction according to the BES database (ISTAT). We expect that, in the regions with positive health conditions (BES indicators at regional level greater than the Italian values) individuals should feel younger since they live better. Table 1 shows Pearson correlation coefficients between age gap and the indicators.

Table 1 – *Correlation coefficients between mean age gap and BES indicators. Signif. codes: p-value ≤ 0.001 (***); (**) 0.001<p-value≤ 0.01; (*) 0.01<p-value≤0.05; (·) 0.05<p-value ≤ 0.1;(-) 0.1<p-value≤1.*

	Female		Male	
	2011	2018	2011	2018
Healthy life expectancy	-0.13 (-)	0.63 (**)	0.39 (.)	0.51 (*)
Personal life satisfaction	-0.07 (-)	0.59 (**)	0.23 (-)	0.50 (*)

L-RaG is related to sentiment indicators and especially to healthy life expectancy (higher and significant correlations). For this reason, we report only graphs showing regional mean age gaps associated with the difference between the values of healthy life indicator at the regional level and the Italian mean (Figure 4). Node colors refer to the pertinent macro-area: north-west (orange), north-east (light orange), centre (green), south (light blue) and islands (violet).

Figure 4 – *Sentiment indicator and mean age gap at regional level.*



According to Figure 4, regions belonging to the first quadrant represent the group associated with the best living conditions in terms of both perceived age and future perspective of healthy life expectancy. In contrast, regions in the third quadrant represent the group associated with the worst living conditions.

The situation for females changed from 2011 to 2018. In fact, the most recent year is characterized by a clear separation of regions into the first and third quadrants. For males, the two years seem more similar.

4. Conclusion

Mortality is a complex phenomenon affected by many factors such as wellness and healthy lifestyle, geographical area, or sex. However, almost all parametric mortality models assume the mortality rate to be function only of chronological age, neglecting the influence of qualitative, non-measurable factors. Based on these considerations, we hypothesize a relationship between mortality and well-being.

Assuming the mortality described by a GM model, we compute the L-RaG age for males and females among Italian regions for the years 2011 and 2018 and the gap between L-RaG and chronological age. Our empirical findings show that the age gap, especially for females, is highly correlated with a healthy life expectancy, supporting our assumption that sentiment indicators based on surveys may be reflected in indicators based on observed data. Moreover, a joint analysis of the age gap and healthy life expectancy allows Italian regions to be ranked based on health conditions.

References

COLLET D. 2003. *Modelling survival data in medical research*. Florida: Chapman & Hall.

DELLAPORTAS P., SMITH A. F., STAVROPOULOS P. 2001. Bayesian analysis of mortality data, *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, Vol. 164, No. 2, pp. 275-291.

GOMPERTZ B. 1825. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies, *Philosophical transactions of the Royal Society of London*, Vol. 115, pp. 513–583.

HELIGMAN L., POLLARD J. H. 1980. The age pattern of mortality, *Journal of the Institute of Actuaries*, Vol. 107, No.1, pp. 49-80.

KAPLAN E. L., MEIER P. 1958. Nonparametric estimation from incomplete observations, *Journal of the American Statistical Association*, Vol. 53, No.282, pp. 457-481.

KOSTAKI A. 1992. A nine-parameter version of the Heligman-Pollard formula, *Mathematical Population Studies*, Vol. 3, No.4, pp. 277-288.

MAKEHAM W. M. 1867. On the law of mortality, *Journal of the Institute of Actuaries*, Vol. 13, pp. 325-358.

MILEVSKY M. A. 2020. Calibrating Gompertz in reverse: What is your longevity-risk-adjusted global age?, *Insurance: Mathematics and Economics*, Vol. 92, pp. 147-161.

NELSON W. 1969. Hazard plotting for incomplete failure data, *Journal of Quality Technology*, Vol.1, No. 1, pp. 27-52.

PETO R., PETO J. 1972. Asymptotically efficient rank invariant test procedures, *Journal of the Royal Statistical Society: Series A (General)*, Vol. 135, No.2, pp.185-198.

RICHARDS S. J. 2020. A Hermite-spline model of post-retirement mortality, *Scandinavian Actuarial Journal*, Vol. 2020, No.2, pp. 110-127.

TABEAU E., WILLEKENS F., VAN POPPEL F. 2002. Parameterisation as a tool in analysing age, period and cohort effects on mortality: a case study of the Netherlands. *In The Life Table*. Dordrecht: Springer.

SUMMARY

Longevity-risk-adjusted global age as a measure of well-being

In this work, we show how the difference between chronological and biological age can be investigated by sentiment indicators constructed using survey data (healthy life expectancy and personal life satisfaction). We invert the GM mortality model to compute longevity-risk-adjusted global age, detecting how an objective indicator (i.e., L-RaG age) mimics a sentiment indicator, thereby implying that human feeling relies on different lifestyle conditions.

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